ST Math

Spatial-Temporal Math:

Underlying Scientific Concepts and Mechanisms

MIND RESEARCH INSTITUTE

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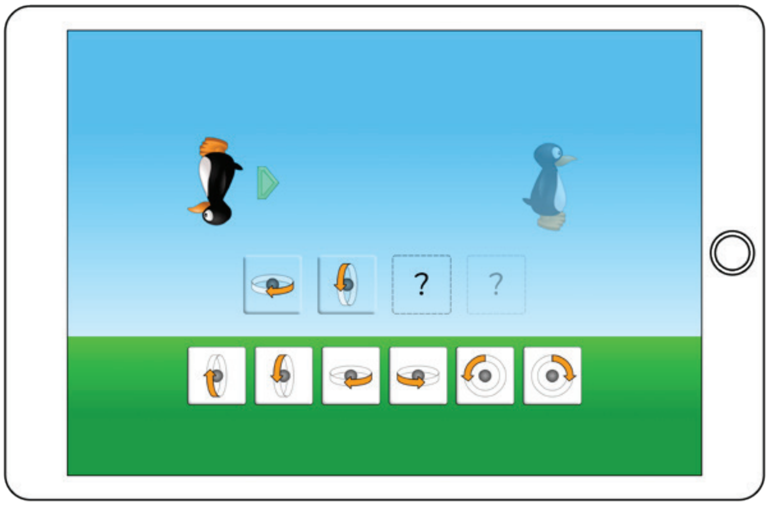
Background

In the United States, student math performance is disturbingly low – only 34% of 8th grade students performed at 'proficient' or above in math on the 2019 National Association for Educational Progress (NAEP) Assessment (National Center for Education Statistics, 2019). 20% of Hispanic and 14% of African American 8th grade students performed on average at proficient or above and 18% of low-income students performed at proficient or above on this assessment. As advanced math skills are vital to our country's increasingly innovation-driven workforce, the low math performance by today's students will have long-term impacts on the economic stability of individuals, communities, and business operations nationwide. Therefore, the development of a solid foundation in math is vital for students to succeed both in school and in the 21st century workplace and it is MIND Research Institute's mission to help students to create this foundation.

Introduction



ST Math is a supplemental, instructional math program created by MIND Research Institute that is currently benefiting more than 1.3 million students in the United States, of which nearly 70% are from traditionally underserved backgrounds. The purpose of ST Math is to increase math knowledge in grades preK-8 in a variety of implementations, including a hybrid in-school/at-home model and full distance learning. Our technique consists of language-independent, animated representations of math concepts presented through a series of puzzles that make learning math fun and engaging. ST Math does not implement a 'teaching by telling' approach, but instead facilitates a 'learning by doing' approach. Self-paced and self-motivating, ST Math provides students with immediate, formative feedback. Math is transformed from something scary and intimidating to something exciting and rewarding, building students' confidence in both their mathematical skills and problem-solving abilities. Our program is proven to assist students regardless of socioeconomic, linguistic or cultural background in becoming mathematical problem solvers, deepening their conceptual understanding and increasing their ability to reason and make mathematical connections.

Summary of Efficacy

MIND has evaluated ST Math at schoolwide levels since 2002, performing more than 100 internal quasi-experimental matched comparison group studies focusing on growth in grade-average mean scale scores and/or focusing on percentage of students exceeding state math test standards (MIND Research Institute, 2020). Besides MIND's internal testing, ST Math has also been independently evaluated (Wendt et al., 2014, 2019). The main outcomes of these evaluations revealed effect sizes of 0.42 and 0.31, respectively, for math scale scores on standardized state assessments. The effect sizes for the percentage of students scoring at or above proficient on these assessments were 0.47 and 0.35, respectively. Notably, the more recent evaluation was carried out across 16 states, including a total of 75 school districts and 474 schools. It has also been demonstrated that an increase in ST Math usage throughout the school year correlated positively with higher math test score gains, which was especially pronounced in students with lower baseline usage (Scherer et al., 2020). It has further been determined that working with ST Math resulted in higher math self-beliefs compared to controls (Rutherford et al., 2020). Finally, there is moderate evidence for ST Math efficacy in grades 3–5 according to Every Student Success Act (ESSA) levels of evidence provided by the U.S. Department of Education guidelines (Boyce et al., 2019).

Concepts and Mechanisms Behind ST Math

ST Math is grounded in experience and supported by science. The current generation of ST Math reflects the continuous development and refinement MIND has incorporated over the last two decades. Figure 1 provides an illustration of the ST Math approach that implements game-based and spatial-temporal learning. In the following, we describe the elements depicted in this figure with the goal of outlining the structure of ST Math as well as the scientific concepts and mechanisms upon which it is built.

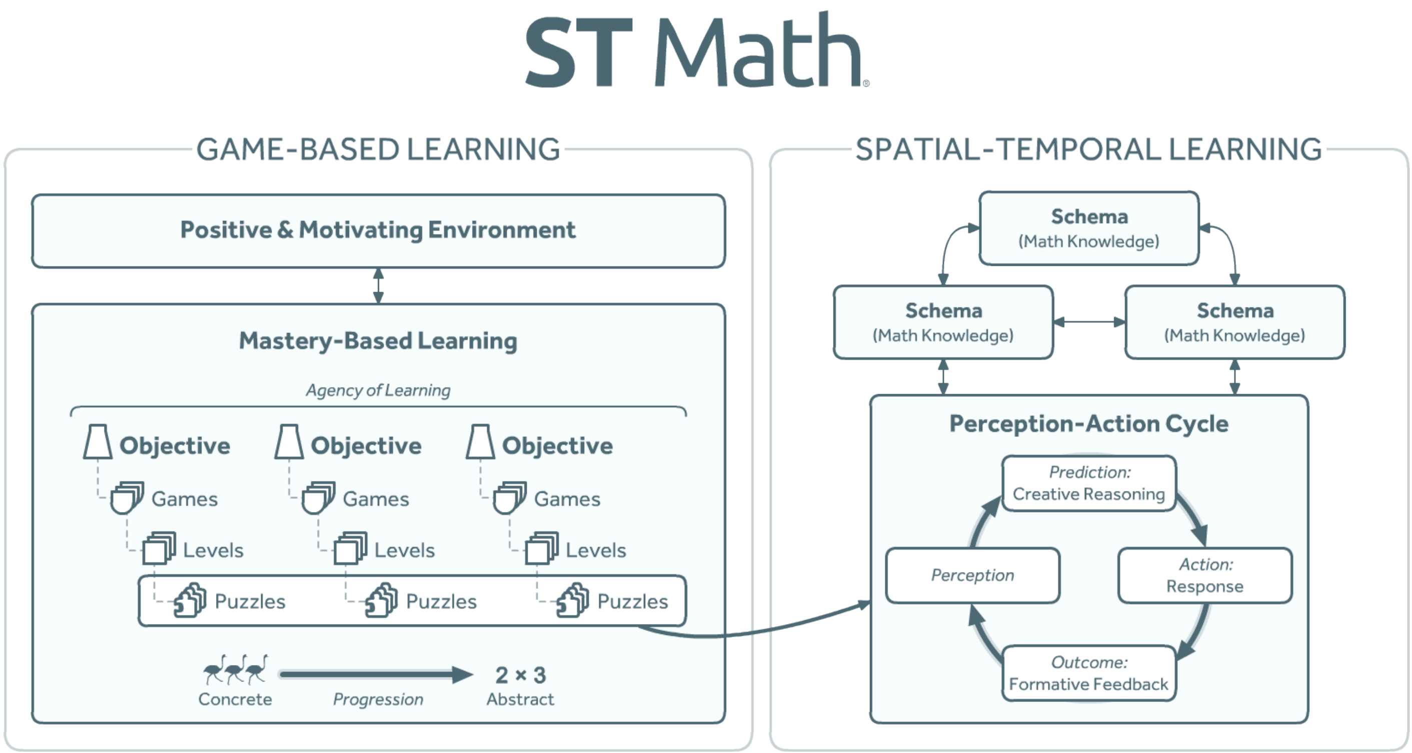


Figure 1. Illustration of ST Math's approach that implements game-based and spatial-temporal learning.

Game-Based Learning

Design Principles

Math learning is unfortunately too often associated with negative connotations such as math anxiety, that is, the negative reaction people experience when they are required to solve a mathematical problem (Maloney et al., 2013). Research investigating how such fear develops suggests that a lack of motivational or cognitive support paired with a high demand for correctness from the teacher can lead to students avoiding math probably because they want to avoid showing incompetence in class (Ashcraft, 2002; Turner et al., 2002). Given math's foundational nature, such avoidance and fear can have detrimental ramifications. In order to provide an inherently positive and motivating math learning environment, the educational content in ST Math is conveyed in a game-like fashion. Games have the potential to be perceived as an enjoyable and fun activity and with that, might provide a preferred means to teach math (Bragg, 2003). In addition, it is conceivable to assume that when students are motivated, they will immerse themselves in an activity and improve their attitude towards the subject over time (Ernest, 1986). Finally, in ST Math, students individually work through the educational content by interacting with the software, minimizing the potential for public embarrassment in math class.

The creation of the game-like activities, through which ST Math conveys its educational content, follows carefully considered design principles (cf., Hirsh-Pasek et al., 2015). First and foremost, all ST Math content is developed with the clear learning goal of improving math knowledge. Further, ST Math games are designed to: (1) help students develop a sense of agency of their own learning; (2) help students engage in the learning process on multiple levels such as behavioral (e.g., be a persistent learner), emotional (e.g., show affection), and cognitive (e.g., remain flexible in problem solving); and (3) provide a meaningful learning experience that is scaffolded and connects to previous knowledge. In general, the game elements of every ST Math game are deeply integrated with the mathematical concept at hand (Figure 2), which is notably different from other existing math games that sometimes simply add a gamification layer on top of a math problem (Figure 3). Such task design aims to reduce distraction by not including unnecessary elements such as animations and feedback that are not content related (cf., Sweller, 2020). Careful consideration is also given to color choices; for example, the colors blue and green are prominently incorporated because of their association with beneficial effects on creativity (Mehta & Zhu, 2009; Studente et al., 2016).

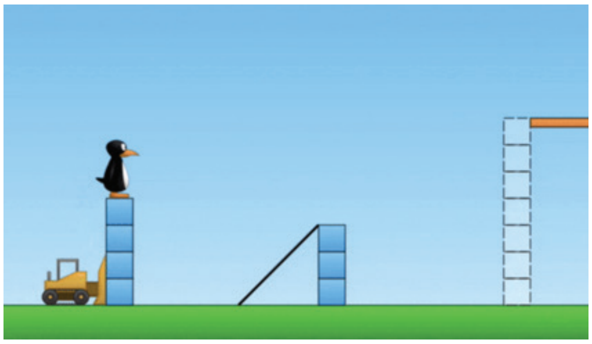


Figure 2. Screenshot taken from the ST Math game Push Box. Given the number of boxes that will be pushed with the forklift (here four) and the stack of boxes behind the ramp (here three), the student is required to determine the sum of these boxes by adjusting the height of the orange platform (here seven as indicated by the dashed lines).JiJi, the penguin, needs to reach the orange platform to clear this particular puzzle. The forklift, the ramp, and Jiji are used to provide the student with formative feedback. Note that all included game elements serve a particular purpose to convey the educational content of interest.

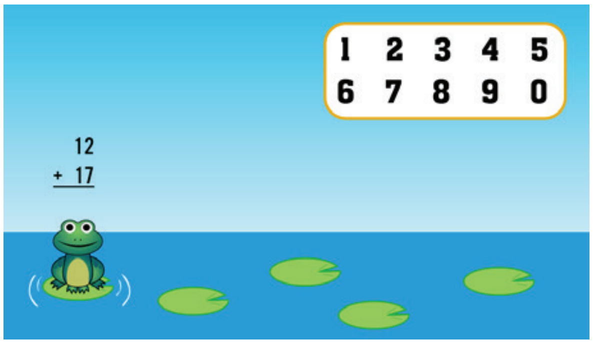


Figure 3. Illustration of a hypothetical game that adds a gamification layer on top of a math problem. The student is required to solve the addition problem to advance the frog to the next lily pad. There is no inherent relationship between the game mechanics and the math content. The math problem could be replaced by any fact-based question outside the realm of math and the game would still function in the same manner.

Mastery-Based Learning

Students often differ in their baseline knowledge and their rate of learning. To take such differences into account, ST Math utilizes a mastery-based learning approach. ST Math is aligned to state and grade level standards and each grade level curriculum consists of 20-30 learning objectives. Learning objectives are composed of games, which in turn are composed of levels that consist of puzzles (Figure 4). Every level comprises a set of puzzles, randomly chosen from a larger set of similar puzzles, that are presented in a semi-random order to the students. If a learner incorrectly solves a number of puzzles the level is restarted with a new set of randomly chosen puzzles. This makes it practically impossible for learners to clear a level simply by memorizing the puzzle solutions, but instead fosters and requires understanding of the underlying math.

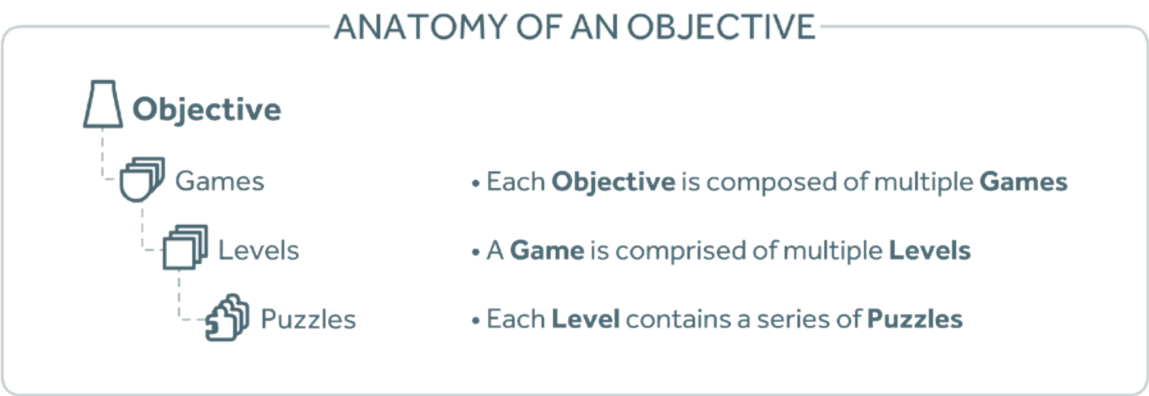


Figure 4. Organization of learning objectives within ST Math. Objectives are composed of different games that are each comprised of different levels and every level in turn consists of multiple puzzles.

To progress, students are required to master a level by solving its puzzles and with that, demonstrate they mastered the concepts presented within a level. While proceeding within ST Math, students interact with progressively more complex levels and more complex games that correspond, in general, with more difficult math principles. The learning pace across games and levels is determined for each student individually and educators are provided with detailed data about student progress, making ST Math an ideal platform to support personalized learning approaches (cf., Pane et al., 2015).

ST Math implements a mastery-based approach which is characterized by focusing on developing an individual's abilities and an emphasis on mastering new skills and overcoming challenges (Meece et al., 2006). Students develop a sense of accomplishment by overcoming challenges and realizing self improvement. This is in contrast to a performance goal orientation that focuses on an individual's performance compared to others where success is defined as being better than others and by meeting or exceeding normative performance standards. A mastery goal orientation results in more positive motivation and learning patterns that are characterized through high levels of effort and persistence and the usage of effective learning strategies that improve mathematical understanding (Meece et al., 2006). To optimally support students' learning and account for a broad range in existing content knowledge, every game within a learning objective starts with relatively simple levels and puzzles that gradually increase in complexity and become increasingly more difficult.

Learning Progression

ST Math follows a learning progression that starts with more concrete problems that gradually become more abstract. This shift coincides with a transition from non-symbolic to symbolic math, leveraging the beneficial impact of manipulatives (Carbonneau et al., 2013). Specifically, ST Math implements digital manipulatives with the goal to support the development of abstract reasoning especially for learners who depend more strongly on familiar objects to extract meaning (cf., Tran et al., 2017). After learning a mathematical concept using digital manipulatives, students then encounter similar problems in more symbolic representations. This approach strategically seeks to foster the discovery of abstract principles using familiar (digital) objects and it helps students to realize that (digital) manipulatives represent a math concept and an object simultaneously (Uttal et al., 1997).

Let us consider the concept of multiplication as an example. In one of ST Math's games, students are presented with creatures that differ in the number of legs they have, such as birds (two legs), robots (three legs), dogs (four legs), or octopuses (eight legs). Focusing on the concept of equal groups, the goal of the game is to assign the correct number of shoes to the creatures (Figure 5). As students progress through the content, the initial objects, i.e., the creatures, are gradually replaced by symbols, until students are finally presented with symbolically represented problems only.

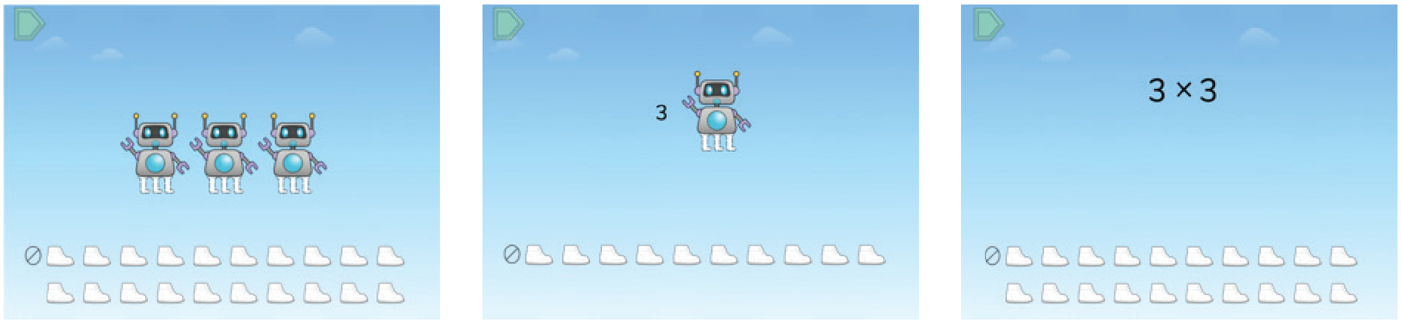


Figure 5. Screenshots taken from the ST Math game How Many Legs. Students start with puzzles that are presented in a non-symbolic way (left panel). As the student progresses through the game, the puzzles substitute non-symbolic elements with more and more symbolic information (middle and right panel). After a solution attempt, feedback is provided by transforming the symbolic information in the right panel back into the non-symbolic information seen in the left panel, visualizing the link between these puzzle elements.

Spatial-Temporal Learning

The 'ST' in ST Math stands for 'spatial-temporal' and refers to the signature design approach of the program's platform. The content within ST Math is designed to express the involved math using space and time, reflecting the fact that core mathematical principles transcend language. ST Math aims to convey mathematical concepts and relatively complex mathematical procedures in an instructive, visual, and appealing way that allows students to develop an intuitive understanding of the involved math (Figure 6). In the following, we describe ST Math's spatial-temporal approach in more detail, focusing on its pivotal aspects that are comprised of schema building, formative feedback, and creative reasoning.

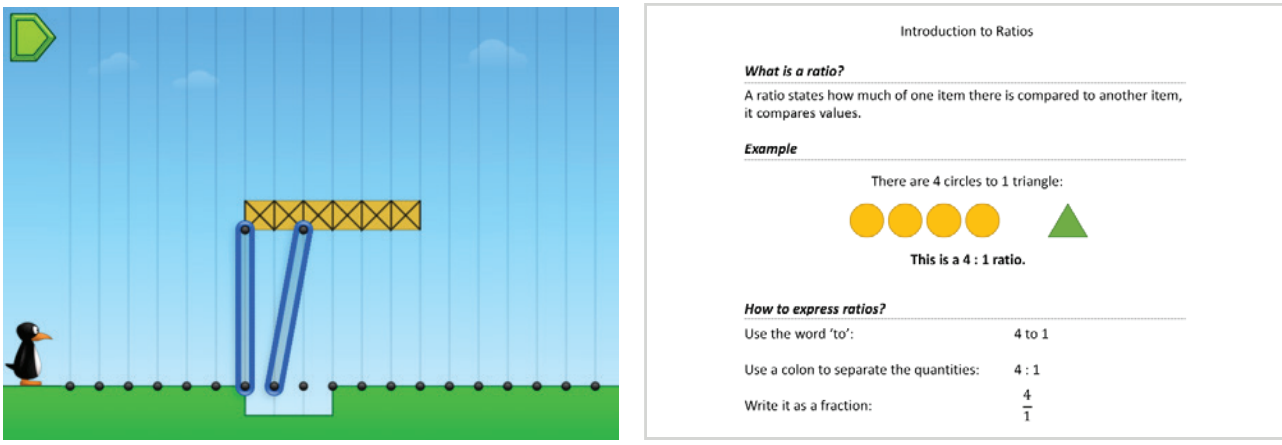


Figure 6. The left panel shows a screenshot taken from the ST Math game Stretch-A-Block. The goal of the game is to fill a hole in the ground with a block that is of a different size than the hole. Students need to find the ratio by which the block must be transformed so that it fits into the hole. This is accomplished by connecting rubber bands attached to the block with pins that are located on the ground, allowing students to determine the ratio that is required to change the size of the block. The right panel shows material from a standard lesson about ratios. In contrast to ST Math's approach, students are instructed in the concept of a ratio using language and are then shown a method for solving corresponding problems.

Schema Building

ST Math promotes the creation and refinement of math schemas. A schema can be conceived as an abstract, mental representation that describes an object, a string of events, or an idea, and essentially anything that can be experienced (Lewis & Durrant, 2011; van Kesteren et al., 2012). An initial schema is formed through implicitly associating and excluding features or attributes. For example, an initial (and not very accurate) schema for the number 1 could be driven through the association of the notion of 'one entity' and the object 'apple.' Schemas are built through experience and they are constantly changing. The just mentioned number 1 schema could undergo refinement in that the 'apple' association is dropped and a written representation ('1') is associated with the notion of 'one entity'. In this sense, schemas are comprised of units (of information) and the relationships among these units. It is not only features that are (or are not) associated with a schema, schemas themselves can also be connected with each other (Rumelhart & Ortoney, 1977). For example, a multiplication schema can be associated with an addition schema because a multiplication problem, such as 3 x 2, can also be written as an addition problem

(2 + 2 + 2). An important property of a schema is that it allows for rapid information processing without requiring conscious and time-consuming thoughts (cf., Ghosh & Gilboa, 2014). For example, a person having a well-developed schema for linear equations does not need to think deeply about it but knows that a line is defined by its slope and its intercept and that the slope can be negative, zero, or positive. Looking at this from a slightly different perspective, it can also be stated that over time, there is a shift from slow, effortful, and conscious processing to quick, effortless, and automatic processing (e.g., W. Schneider & Chein, 2003). This shift reflects the fact that learning something new in the beginning often requires deliberate attention and conscious processing which can gradually decrease as the proficiency with the topic grows.

A new schema that is not very well developed can be described as weak. Through experience and associated refinement, it eventually becomes a strong schema. This transition represents the development of knowledge. When it comes to math, different types of knowledge are often distinguished, such as conceptual or procedural knowledge (e.g., Rittle-Johnson & M. Schneider, 2015; Star, 2005)[1]. While the former can be defined as knowledge about facts, concepts, and principles, the latter refers to actions and manipulations for completing a task (de Jong & Ferguson-Hessler, 1996). The knowledge base of a learner can be described for each of these knowledge types and has been termed knowledge quality. de Jong & Ferguson-Hessler (1996) note that the quality of a knowledge type can be expressed in multiple ways. For example, one can describe quality based on the level of knowledge (superficial vs. deep), based on the structure (isolated features vs. meaningful structure), based on the degree of automation (conscious vs. automatic processing), or based on the degree of generality (domain specific vs. general). Accordingly, quality can be expressed through the connectedness within the knowledge type, the completeness of the knowledge, its structure and its generality of application, as well as the connectedness between the different knowledge types (Baroody et al., 2007). Denoting a lower quality of knowledge, a weak schema represents routine expertise that can be applied in a narrow and not logical manner, is characterized as being unconnected and context-specific, and allows a learner to apply their knowledge to familiar but not new tasks (Baroody et al., 2004, 2007). In contrast, a strong schema represents adaptive expertise that can be applied broadly and logically, is characterized as well-connected and general, allowing a learner to apply their knowledge creatively and flexibly to familiar and new tasks (Baroody et al., 2004, 2007). Deep mathematical understanding is represented in the form of very strong, refined, and integrated schemas that represent an overarching concept that connects multiple concepts and procedures within or across domains or topics (cf., Baroody et al., 2004). Such deep understanding provides the basis for understanding different concepts. For instance, the concept of equal partitioning is important to understanding a host of other concepts including but not limited to division, fractions, and measurement (i.e., dividing a continuous quantity in equal, countable parts). Deep understanding will also allow learners to relate these concepts with each other, within and across domains. Instead of perceiving mathematical knowledge as a set of isolated definitions and procedures, learners recognize mathematical knowledge as structured and/or cohesive. A deep understanding also provides a rationale for different procedures and enables a learner to understand and invent algorithms (Hiebert, 2003), and ultimately allows the learner to transition mathematical knowledge to everyday situations and applications (Baroody et al., 2007).

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[1] Other types of knowledge have been suggested. For example, *situational knowledge* refers to knowledge of domain-specific situations that allows extracting relevant features and supplement provided information if required (de Jong & Ferguson-Hessler, 1996). The discussion of different types of knowledge is beyond the scope of the current topic and the focus remains on conceptual and procedural knowledge.

The way ST Math supports the development of math-related schemas can readily be described with the concept of a perception-action cycle that originated from neuroscientific research (Fuster, 2004). The cycle describes learning as a feedback loop that consists of (1) a prediction guided by existing knowledge and the perception of the environment, which is followed by (2) an action through the learner, which results in (3) an outcome that is (4) perceived by the learner. This cycle is represented in Figure 1, supplemented with labels of topics discussed in the following to provide a schematic overview of ST Math's spatial-temporal learning approach. At the end of the cycle, the learner compares the perceived outcome with the initial prediction and based on that comparison, the knowledge base of the learner changes. It is through the constant interaction of an individual with the environment (i.e., ST Math) that learning happens: through the integration of previous experiences and novel sensory information (i.e., the feedback provided by ST Math), the learner creates an expectation (through creative reasoning, discussed below) and compares it with the result of a performed action (i.e., the response of a learner) which then modifies and refines existing knowledge. This perception-action cycle is continuous, resulting in creations and modifications of schemas. For example, a young student interacting with base-10 blocks might predict how many blocks will be left in an initial stack after a certain number of blocks is removed from it. This experience is either in line or not with the student's expectations and will either reinforce or adjust the existing schema for subtraction, which then leads to a modified prediction in a next attempt, and so on.

When it comes to interacting with ST Math, students engage in a similar process. They are presented with a problem, for which they may only have a crude understanding initially. The student will make a prediction of what the solution could be and ST Math then provides feedback as a result of the student's response. This allows the student to refine the initial answer choice in a next attempt, which is again followed by feedback provided through ST Math, and so on. Going through such cycles, the student can refine and improve existing math-related schemas. However, in order to be able to improve at all, it is of crucial importance what kind of outcome the student perceives. This perceived outcome is provided by ST Math as formative feedback and represents a pivotal cornerstone of ST Math.

Formative Feedback

The type of feedback students receive is crucial for learning (Shute, 2008; Van der Kleij et al., 2011, 2015). Broadly speaking, feedback can be described as being confirmatory or elaborative (Kim et al., 2018). Whereas confirmatory feedback entails limited information mainly explaining whether a response was correct or not and sometimes also includes the correct answer, elaborative feedback provides more detailed information why a response was correct or not, adding an important instructive element to the response the learner receives (cf., Hattie & Timperley, 2007). It is worthwhile to point out that elaborative feedback constitutes an important part of formative assessments (Sadler, 1989). In contrast to summative assessments that aim to measure student learning typically at the end of an instructional unit, formative assessments focus on improving learning (Bennett, 2011). For that purpose, formative assessment typically not only involves an assessment component that evaluates a student, but also a formative feedback component that is supposed to lead to improved learning (Sadler, 1989; Shute, 2008). Accordingly, the elaborated feedback provided by ST Math is called formative feedback[2], which is defined as the

[2] In selected cases elaborative feedback is also called *informative feedback* (e.g., Kim et al., 2018), a term that we sometimes also use in describing the feedback ST Math provides.

information conveyed "to the learner with the intention to modify his or her thinking or behavior for the purpose of improving learning" (Shute, 2008, p. 154). It has been demonstrated that formative feedback leads to better learning outcomes than confirmatory feedback, confirming the feedback approach implemented in ST Math (Van der Kleij et al., 2015).

The formative feedback implemented in ST Math is distinctly different compared to other educational approaches and is rooted in game-based learning. When designed well, video games allow players to instantly observe the consequences of their actions to learn game rules and develop conceptual strategies required to win the game. ST Math's visual approach allows accessing this conceptual feedback mechanism by presenting mathematical problems in the form of interactive puzzles where the mechanics of the puzzles are visualizations of the underlying mathematics (Peterson & Bodner, 2009). Students do not simply select answers and receive feedback only on their accuracy, but instead use interactive game elements to construct solutions which are then animated, allowing them to observe and learn how each element contributes to the puzzle solution. This formative feedback allows students to modify their thinking and behavior as well as update and refine their schemas, strategies, and concepts (Shute, 2008).

The appropriately increasing difficulty levels of the problems within ST Math in combination with the formative feedback are assumed to result in a positive impact on student's self belief (Bandura, 1986). Such an increase in math self beliefs has recently been empirically confirmed in ST Math (Rutherford et al., 2020). There is evidence that self beliefs are positively linked to achievement (e.g., Honicke & Broadbent, 2016), which can, for example, be explained through prolonged persistence on problem-solving tasks found in students with high self efficacy (Bouffard-Bouchard et al., 1991). This underlines the importance of the positive change in self efficacy ST Math is able to induce.

In regards to the learning progression within ST Math, another goal of the formative feedback is to help students master the most difficult problems of the program. With that, formative feedback within ST Math fulfills another important motivational role by helping learners to realize that their abilities and skills can be improved through practice, that mistakes are part of acquiring skills, and that effort is important to get better at anything (Hoska, 1993). Importantly, the spatial-temporal approach and the associated schema development in ST Math emphasize creative reasoning as a student's independent process to learn and discover the math concepts at hand.

Creative Reasoning

A dominant teaching method in the United States is to present students with a problem and provide an adequate solution method for it, which is then followed-up by extensive practice that makes use of this method (Hiebert, 1999, 2003). Relying on such an approach is understandable and seems appropriate because teaching of pre-defined solution methods (or algorithms) saves time, prevents miscalculations, and a substantial majority of the tasks in textbooks can be solved with them (Jonsson et al., 2014, 2016; Norqvist, 2018). Although it is beneficial to know about specific algorithms, merely presenting predefined solution methods does not necessarily encourage the development of deeper understanding of math principles and students are at risk to use such solutions in an unreflected and superficial way without any conceptual understanding of them (Hiebert, 1999, 2003; Jonsson et al., 2016). Therefore, algorithm-based teaching approaches might have a limited impact on students' long-term math abilities development (Carpenter et al., 1998; Hiebert & Wearne, 1996). However, it has also been shown that the development of procedure-based and conceptual knowledge is bidirectional, supporting the notion that focusing on only one of the two knowledge types is undesirable (M. Schneider et al., 2011).

To promote the development of deep or conceptual math understanding, ST Math does not reveal problem solutions to students but instead makes students the creators of their own knowledge and fosters the development of algorithms by the students themselves. The formative feedback guides students to the same standard algorithms and precise mathematical definitions as a textbook approach, but instead of deliberately defining algorithms, the games guide students to build understanding of mathematical concepts through creative reasoning. Every ST Math game students encounter for the first time requires them to solve problems without having any prior knowledge about them. Besides applying such problem-solving skills, students must also be able to reason by justifying their choices and conclusions, and they also need to take into account their existing conceptual math understanding as well as the formative feedback provided by ST Math. An important aspect of growth in conceptual understanding through creative reasoning is the cognitive effort that students need to expend (Hiebert & Grouws, 2007). This effort, also termed struggle, refers to making sense of mathematical problems that are just within students' reach of understanding as opposed to simply memorizing a presented solution method (Hiebert & Grouws, 2007). Struggle requires students to work more actively and effortfully to make sense of a situation, which then leads to the formation of better connections to knowledge they already possess (Hiebert & Grouws, 2007). It makes intuitive sense that the struggle must be productive for students, that is, the problems they are facing must be challenging but not insurmountable, a notion that Vygotsky (1978) described as the zone of proximal development. In order for a problem to optimally foster learning, its difficulty must be desirable, meaning that the learner must have the required existing knowledge and skill to respond successfully (Bjork & Bjork, 2014). In this sense, ST Math implements several features that promote a student's learning as best as possible and keeps the focus on the relevant math concept. For example, every game is designed with the intention that students can focus as much as possible on the math of interest without a need to think about the game mechanics. Further, the difficulties of the puzzles are carefully scaffolded, starting with relatively simple problems that become increasingly complex as the student progresses and ultimately lead a student to mastering a game and its levels.

Further confirming ST Math's learning method, a set of recent experiments demonstrated the advantage of creative reasoning over algorithm-based approaches. For example, Jonsson et al. (2014) compared a group of students either practicing with a creative reasoning approach or an algorithm-based approach. Their results showed that one week after practice, the creative reasoning group outperformed the other group on tasks that required the re-construction of the practiced solutions. The authors assumed that the struggle the creative reasoning group had to go through resulted in a deeper memory trace that students could benefit from and that the self-generation of solution methods facilitated conceptual understanding of the specific task solving methods. These results were confirmed in a follow-up study that also investigated brain activations (Karlsson Wirebring et al., 2015). In addition to the replicated math results, the lower activity in certain brain areas in the creative reasoning group (i.e., an area called the left angular gyrus) suggested easier memory access to the solution method. A further study emphasized that it is mainly the effortful struggle that seems to drive the superior performance in favor of the creative reasoning group (Jonsson et al., 2016). Finally, a study by Norqvist (2018) considered the fact that teachers and textbooks sometimes provide conceptual explanations along with algorithmic solution approaches and compared such a group with a group that practiced using a creative reasoning approach. Again, the results revealed that creative reasoning outperforms algorithmic approaches regardless of whether they were accompanied with conceptual explanations or not.

Evolution of ST Math

ST Math combines a multitude of different mechanisms into a comprehensive program (Figure 1) that has been found repeatedly to result in educationally meaningful math improvements (Wendt et al., 2014, 2019). Although many of the mechanisms discussed result in positive effects on learning, science does not provide detailed guidance on how to combine them and how to create a math learning platform that is effective in today's classrooms. Therefore, for a successful platform such as ST Math to evolve, it is imperative to continuously test, validate, and improve the approach. MIND started this work two decades ago, and we continue doing so with the goal to further improve our learning platform that supports students to become better math learners and creative problem solvers.

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